Name:_____

Worksheet 3 - parabolas, circles, shifting graphs, completing the square, distances in the plane

If you had trouble with question 3a or 3b, do these problems.

1. Graph $y - 2 = (x - 1)^2$.

   Method of shifting: Cool fact: $y - b = (x - a)^2$ has same shape as the parabola $y = x^2$, but it has been shifted up by b and to the right by a. The vertex is at (a, b) instead of at $(0, 0)$.

2. Graph $y - 3 = (x - 1)^2$.

3. Graph $y = x^2 - 3x + 2$.

   Method of finding the roots: if the quadratic factors into $(x - a)(x - b)$, we know that the curve crosses the x-axis at $(a, 0)$ and $(b, 0)$. Cool fact: In this case, the x-coordinate of the vertex of the parabola will be halfway between a and b.

4. Graph $y = x^2 - 1$.

5. Graph $y = x^2 + 2x + 3$.

   Method of completing the square: Recall that $(x - a)^2 = x^2 - 2a + a^2$. Use this to rewrite your quadratic equation as $y = (x - a)^2 + b$. Here,

   $$x^2 + 2x + 3 = (x^2 + 2x + 1) + 2 = (x + 1)^2 + 2.$$

   Now use the method of shifting.

6. Graph $y = x^2 + 4x + 5$.

7. Find the center and radius of the circle $y^2 + x^2 - 2x = 0$, and then graph it.

   Cool facts:

   The equation for a circle centered at (0,0) with radius r is $x^2 + y^2 = r^2$.

   The equation for a circle centered at (a, b) with radius r is $(x - a)^2 + (y - b)^2 = r^2$. (Use the method of shifting.)

   Complete the square for both x and y to write the equation in this form, and then read off the center and radius.

8. Find the center and radius of $y^2 - 2y + x^2 + 6x = -9$, and then graph it.

9. Find the distance between a given point (x, y) and the point directly above or below it on the x-axis. I.e., find the distance between the point (x, y) and the point $(x, 0)$.

10. Suppose the curve $y = x^2$, $0 \leq x \leq 2$ is rotated around the line $x = 3$ to get a surface of revolution. What is the radius of the horizontal slice of the surface at height y? I.e., what is the horizontal distance from the point (x, y) on the curve to the point at the same height on the line $x = 3$?