Name:_____

Worksheet 5 - limits and growth rates

If you had trouble with problem 5a or 5b, do the following problems.

1. Find the limits. First try plugging in $x = 0$ to see if there is an easy answer.

   (a) $\lim\limits_{x \to 0^+} \dfrac{1}{x}$

   (b) $\lim\limits_{x \to 0^+} \dfrac{x+1}{x}$

   (c) $\lim\limits_{x \to 0^+} \dfrac{x + e^x}{x^2 + 1}$

   (d) $\lim\limits_{x \to 0^+} \dfrac{x + x^3}{x^2 + 3x^3}$

2. When can you use L'Hospital's rule?

3. If f and g are functions, and positive for sufficiently large x, we say that "$f(x)$ grows faster than $g(x)$ as x goes to infinity" if

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty.$$

   Which of each pair of functions grows faster as x goes to infinity?

   (a) x^2 or x^3 ?

   (b) x^{100} or e^x ?

   (c) $\sqrt{x}$ or $\ln x$?

   (d) $x^{1/100}$ or $\ln x$?

   (e) $x^{1/100}$ or $2 + \sin x$?

4. Find the limits. Trick: Compare the fastest growing term in the numerator to the fastest growing term in the denominator.

   (a) $\lim\limits_{x \to \infty} \dfrac{1}{x}$

   (b) $\lim\limits_{x \to \infty} \dfrac{x+1}{x}$

   (c) $\lim\limits_{x \to \infty} \dfrac{x+1}{x^2 + 1}$

   (d) $\lim\limits_{x \to \infty} \dfrac{x + x^3}{x^2 + 3x^3}$

   (e) $\lim\limits_{x \to \infty} \dfrac{\ln x + x}{\sqrt{x}}$

   (f) $\lim\limits_{x \to \infty} \dfrac{x + x^2}{1 + e^x}$

   (g) $\lim\limits_{x \to \infty} \dfrac{2 + \sin x}{1 + \ln x}$