Name:_____

Worksheet 6 - Definition of the derivative, Fundamental Theorem of Calculus.

If you had trouble with question 6, do the following problems.
For the problems below, assume the car is moving in a straight line.

1. Suppose that the position of a car is given by $f(t)$ for $0 \leq t \leq 10$, where f is given in meters and t is given in seconds.

   (a) Write down an expression for the average velocity of the car between time $t = 1$ and $t = 2$.

   (b) Write down an expression for the average velocity of the car between time $t = 1$ and $t = 1.05$.

   (c) Use limits to write down an expression for the instantaneous velocity of the car at time $t = 1$.

2. Complete this limit definition of the derivative: Given a function f, the derivative of f at a is given by

$$f'(a) = \lim_{h \to 0} \left(\qquad \right)$$

3. Suppose that the velocity of a car is given by $v(t)$ for $0 \leq t \leq 10$, where v is given in meters/second and t is given in seconds.

   (a) Write down an expression to estimate the change in position of the car between time $t = 1$ and $t = 1.05$.

   (b) Write down an expression to estimate the change in position of the car between time $t = 1$ and $t = 2$, where you get to measure the velocity every twentieth of a second.

   (c) Use limits to write down an expression for the actual change in position of the car between $t = 1$ and $t = 2$.

   (d) What do any of these questions have to do with the area under the graph of $v(t)$?

4. Precisely state both parts of the Fundamental Theorem of Calculus.

5. Suppose that water flows into a tank at a RATE of $200 - 4t$ liters per minute, for $0 \leq t \leq 50$. If the tank starts with 1000 liters at time $t = 0$, how much liquid will be in the tank at time $t = 10$?

6. Find the derivative of $f(x) = \int_0^{x^2} UGLYFUNCTION(t)dt$

7. Suppose that the triangle between the lines $x + y = 1$, $y = 0$, and $x = 0$ is rotated about the y-axis to obtain a cone. Find the area of the circular slice of this cone at height y, and find the total volume of the cone.