Name:________________________

Worksheet 10 - integration by substitution

Given functions f amd u, the chain rule says that $\frac{d}{dx}f(u(x)) = f'(u(x))\,u'(x)$. Given the indefinite integral $\int f'(u(x))\,u'(x)\,dx$, we can replace $u'(x)\,dx$ with du to write

$$\int f'(u(x))\,u'(x)\,dx = \int f'(u)du.$$

For a definite integral we must also change the limits of integration:

$$\int_{x=a}^{x=b} f'(u(x))u'(x)\,dx = \int_{u=u(a)}^{u=u(b)} f'(u)du$$

1. Guess $u(x)$ and substitute to compute the integrals. Write your answer in terms of x.

   (a) $\int x\cos(x^2)\,dx$

   (b) $\int \frac{x}{x^2+1}\,dx$

   (c) $\int \frac{\cos(3x)}{\sqrt{\sin(3x)}}\,dx$

2. Sometimes finding a good u requires harder guesswork; choose a u that has $u'(x)$ available, and then see if you can rewrite everything else in terms of u.

   (a) $\int \frac{e^x}{1+e^{2x}}\,dx$

   (b) $\int x\sqrt{2-x}\,dx$

   (c) $\int x^3\sqrt{x^2+1}\,dx$

3. Sometimes you need to split the integral into pieces. For instance, find $\int \frac{x+1}{x^2+1}\,dx$.

4. Be careful to use the correct limits of integration when computing definite integrals.

(a) $\int_0^3 x\cos(x^2)\,dx$

(b) $\int_0^1 x\sqrt{2-x}\,dx$

(c) $\int_2^3 \frac{x}{x^2+1}\,dx$

(d) $\int_0^\pi (\sec(x/4))^2\,dx$

(e) $\int_0^{\frac{1}{38}} \frac{\arcsin(19x)}{\sqrt{1-361x^2}}\,dx$