SARSI 2016

First Week Lectures

Math - Kim Whittlesey

Lecture 5:

Braids, Google Map Space, and the Universe

الضفائر و خرائط جوجل و الكون

Braid groups

Some braids

Some more braids

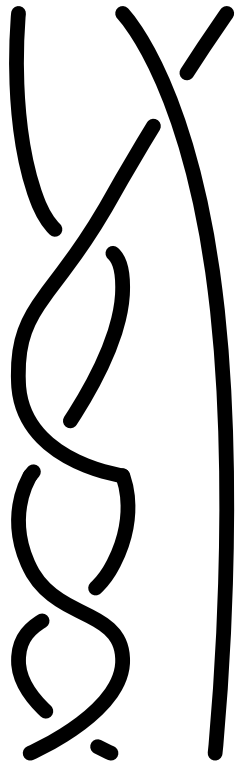
Standard Braid

Trick Braid

3 in 5 mixed Braid

Some more braids

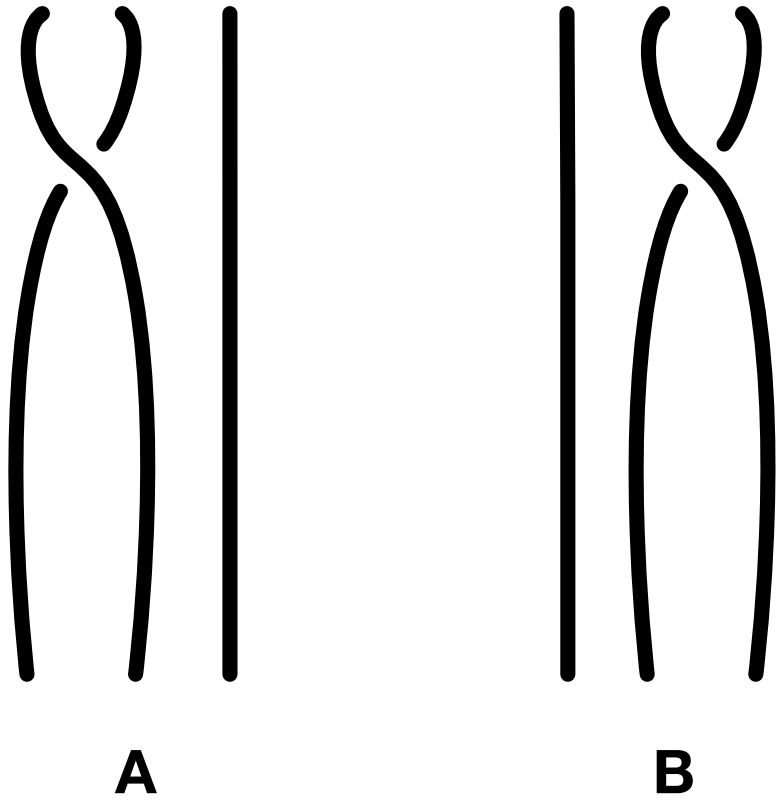
We'll start
with braids
with 3 strands.



Use three cords and a piece of tape to make one of these:

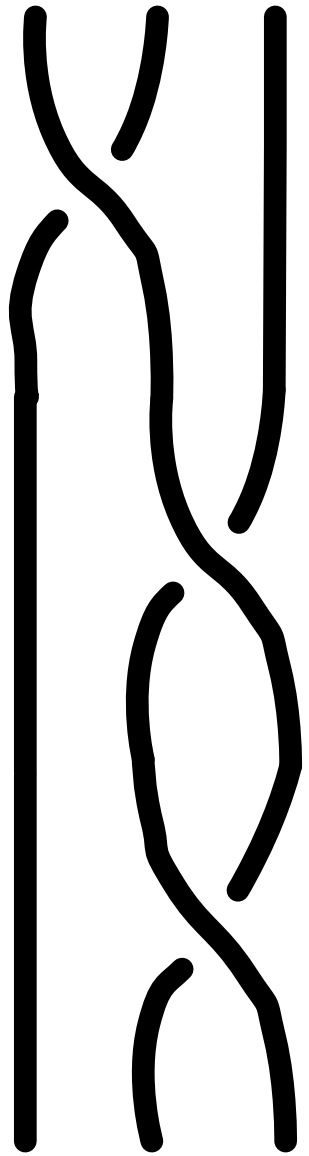
Try making these two braids.

We'll call them A and B.



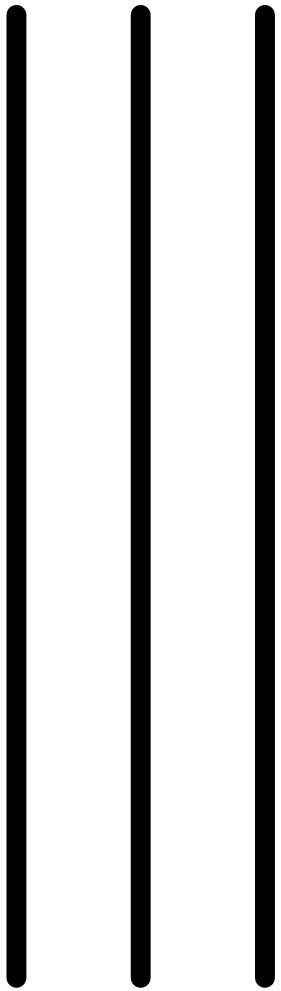
We can
"multiply"
braids by
stacking them.

Here is braid
A*B*B or
ABB

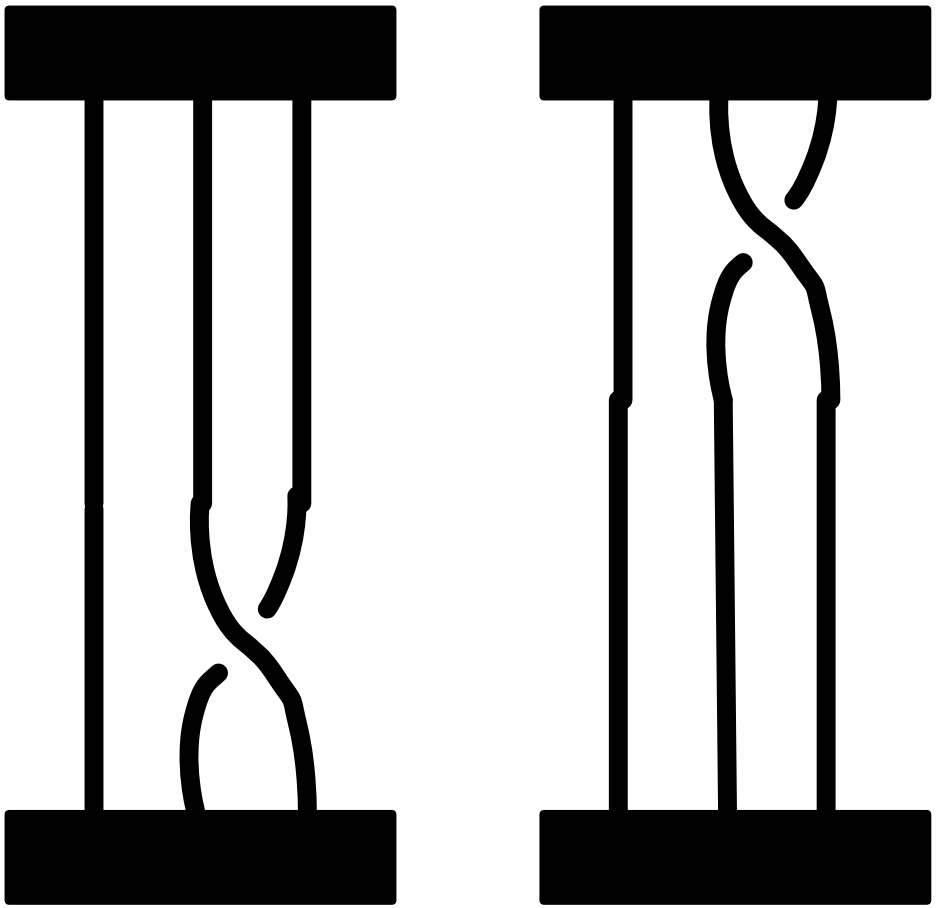


Here is the identity braid.

We'll call it I.



Two braids are the SAME if you can "comb" one of them to other while keeping the ends fixed.



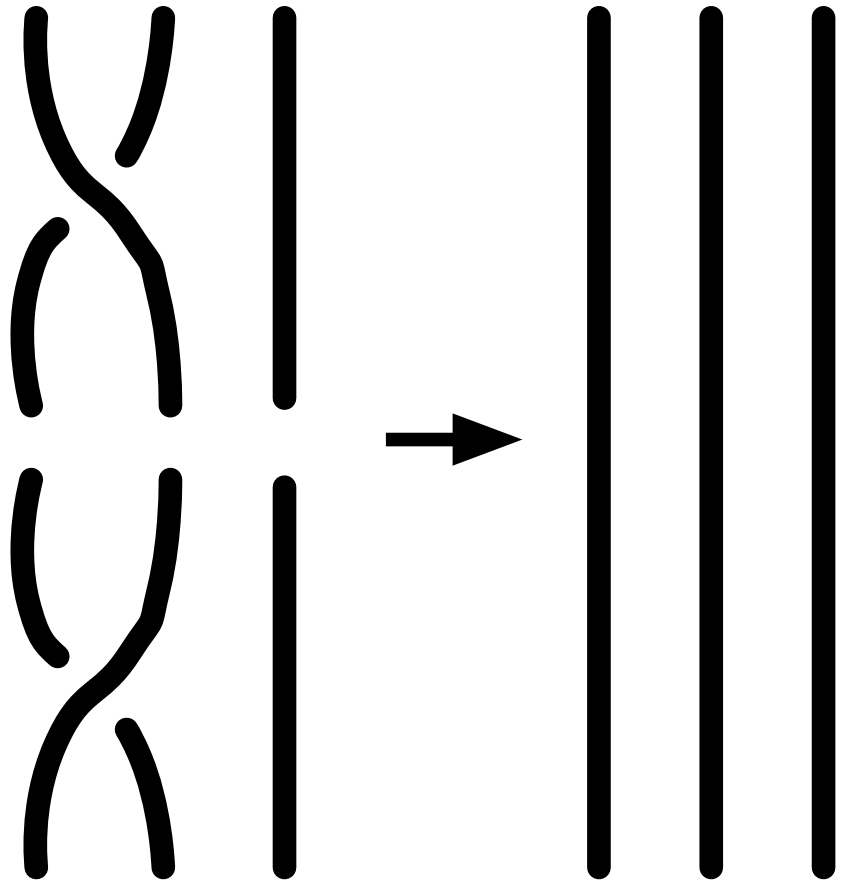
Here are A and

A^{-1}, its inverse.

If you comb

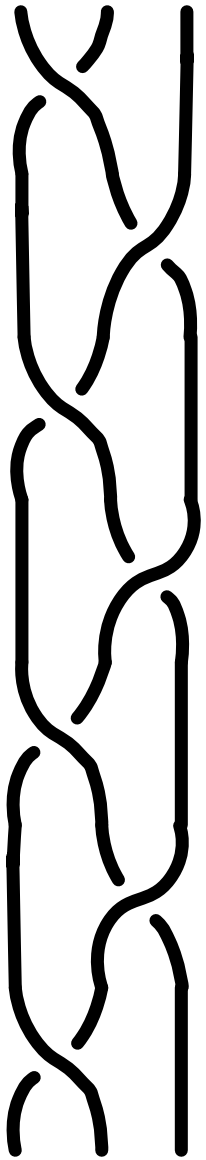
$A^{-1}*A$,

you get I.



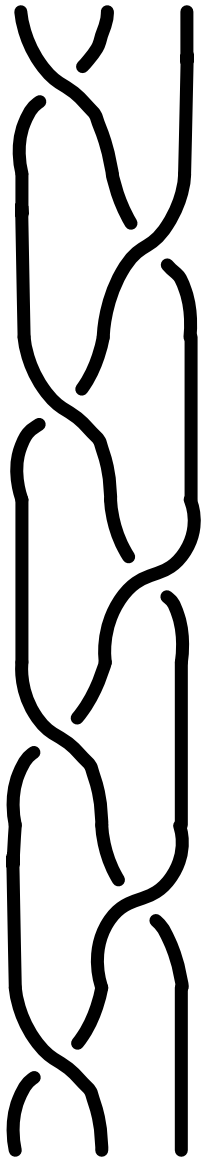
Problem:

What is the pattern for the "usual" braid that you put into hair?



The pattern is

$AB^{-1}AB^{-1}A\ldots$

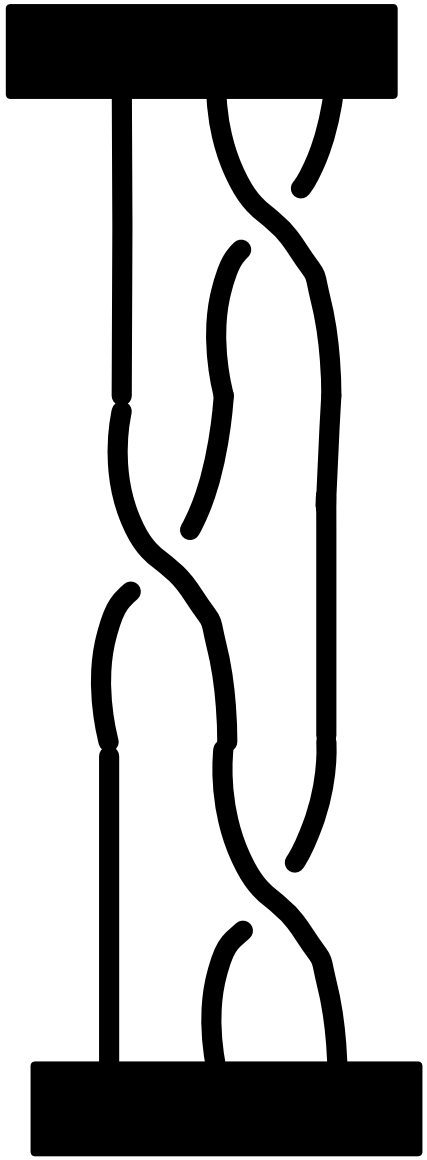


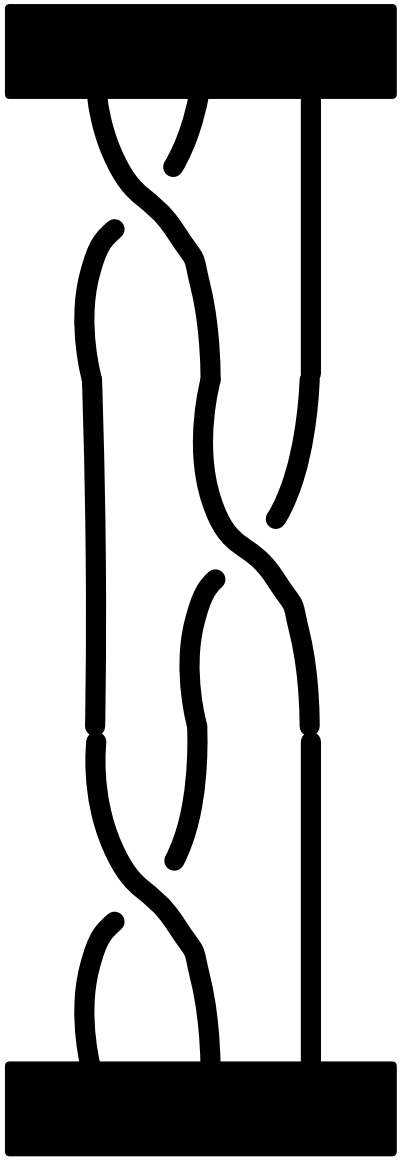
Problem:

Is BAB = ABA?

Use your strings to check.

BAB

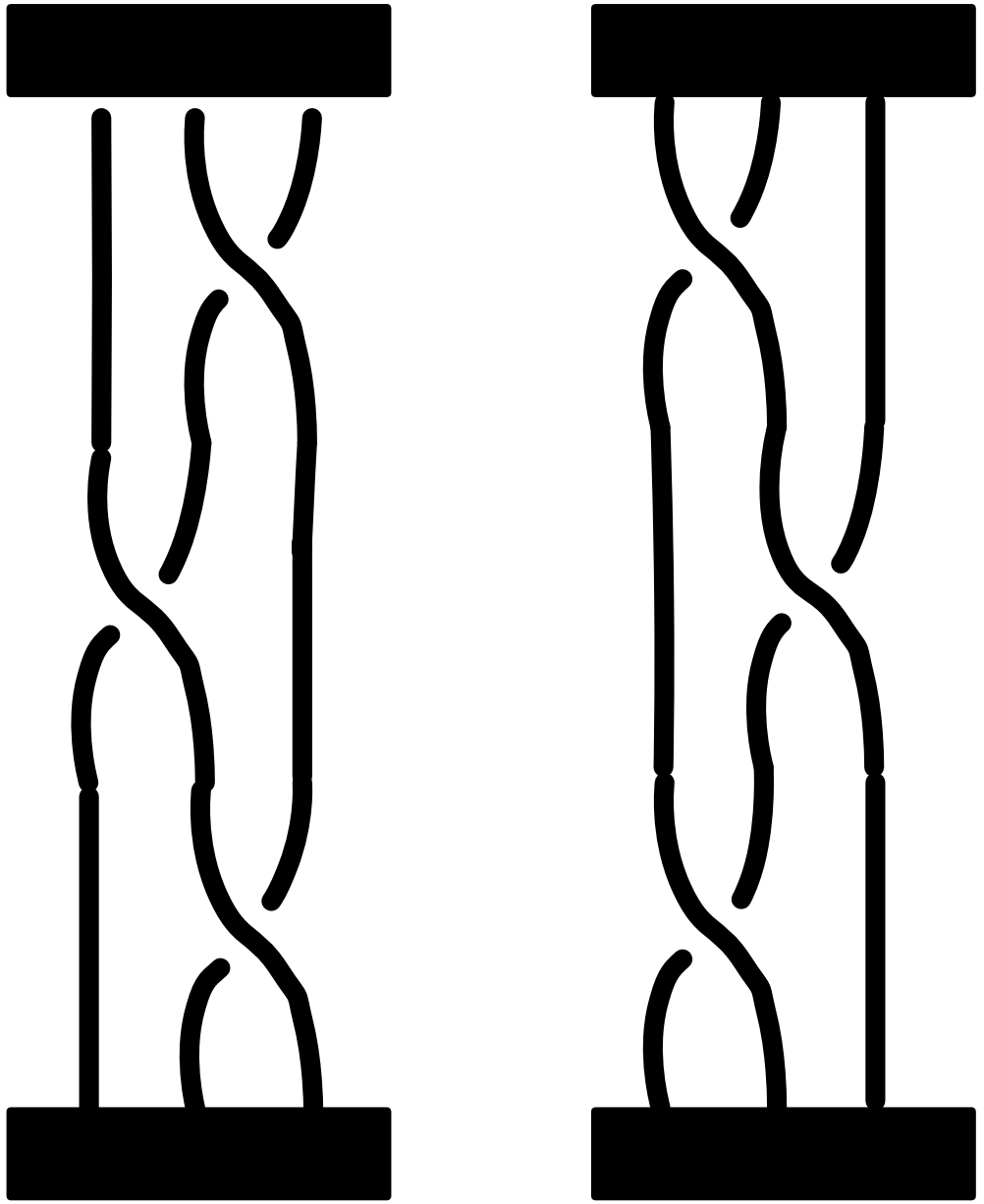




ABA

You can comb the braid BAB so that it becomes ABA.

So, ABA = BAB in the group.

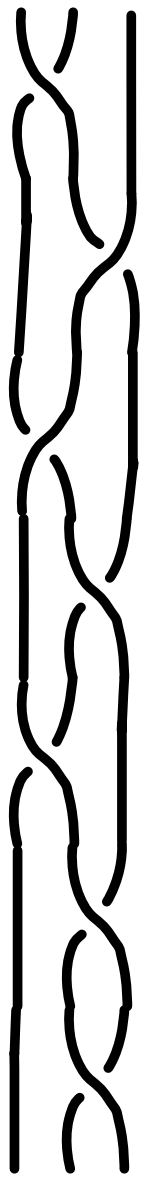


The set of braids, with this kind of "multiplication", forms a GROUP.

Problem:

Simplify this braid as much as possible:

$AB^{-1}A^{-1}BABB$



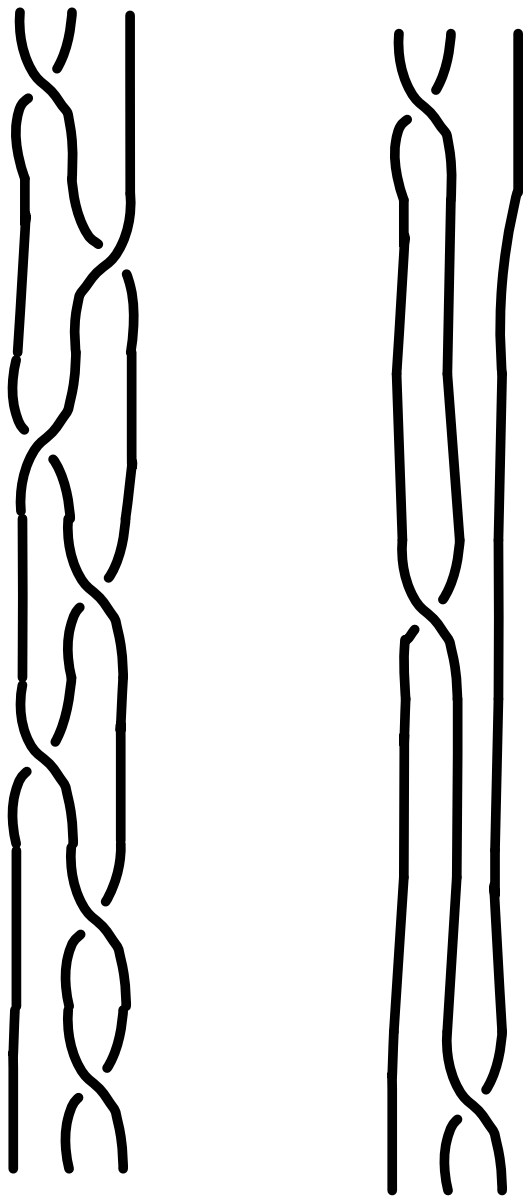
$$AB^{-1}A^{-1}\mathbf{BAB}B$$

$$=$$

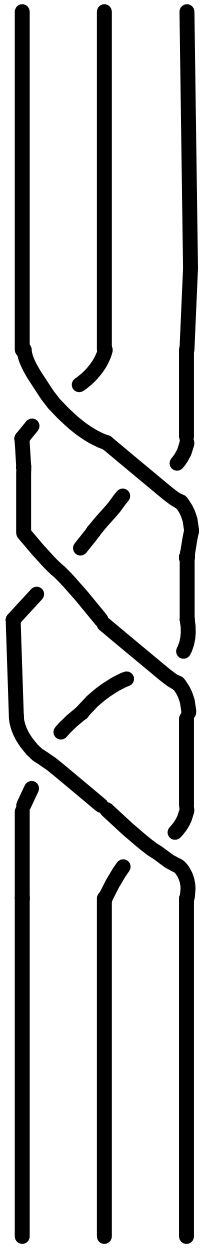
$$AB^{-1}A^{-1}\mathbf{ABA}B$$

$$=$$

$$AAB$$



This is the "full twist" braid, ABABAB.



A full twist on a belt

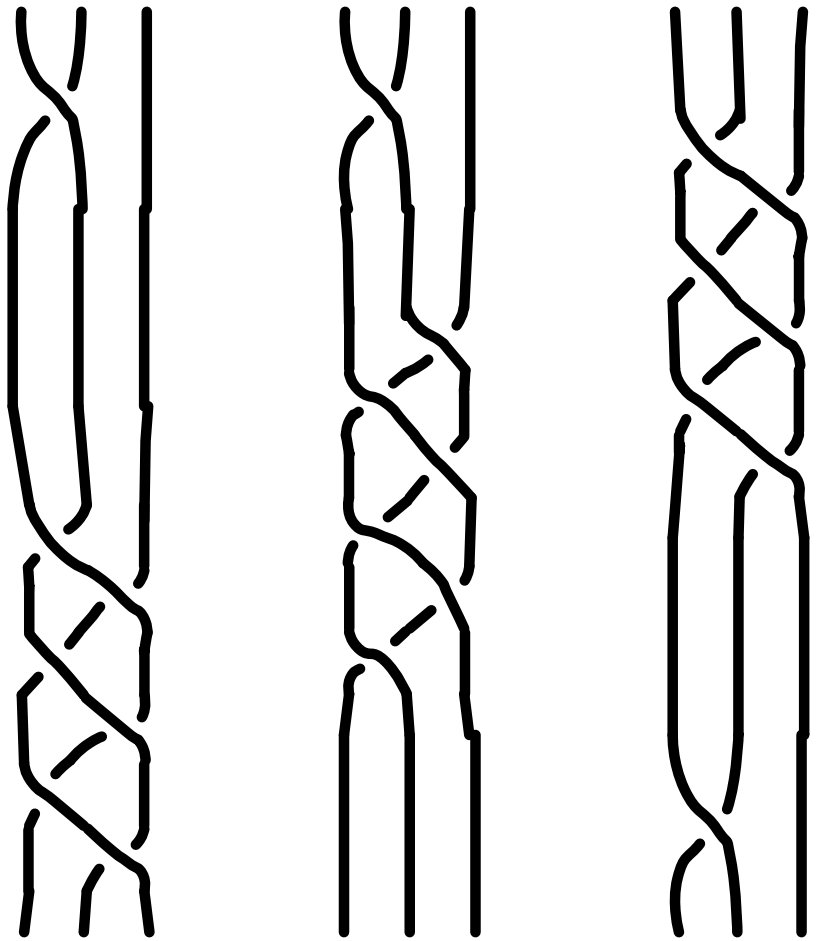
is the same as a loop-de-loop.

True or False:

(ABABAB) A = A (ABABAB)

(ABABAB) B = B (ABABAB)

ABABAB can slide up to the TOP of the braid without changing the rest of the braid.

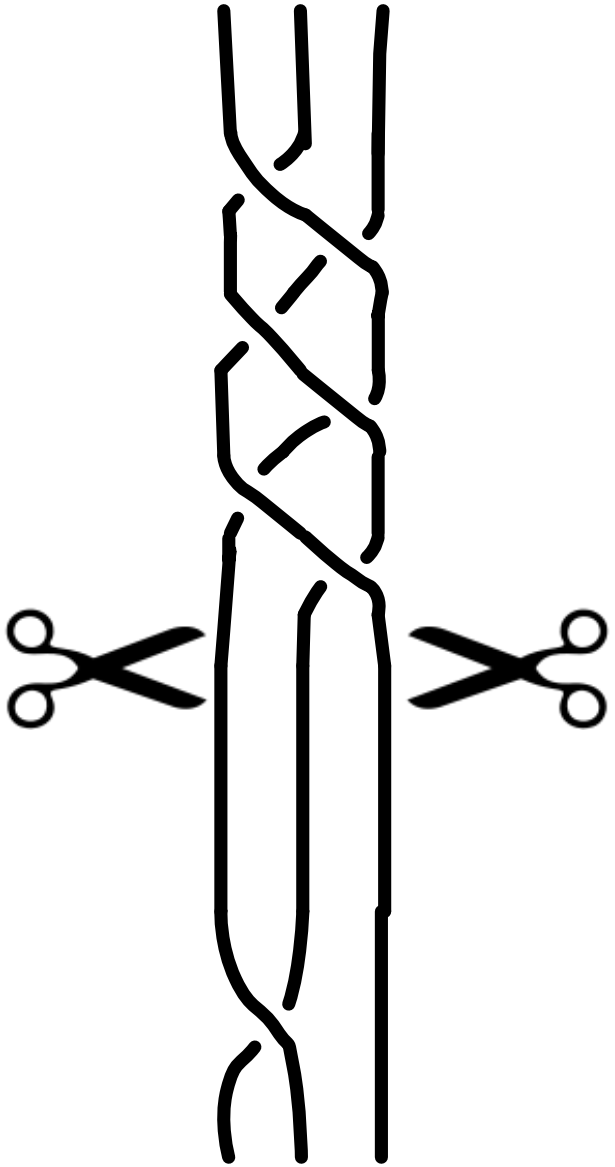


We can modify
the braid group
slightly
by adding the equation
$ABABAB = I$.

Adding

ABABAB = I

is like moving all the full twists to the top, and then cutting them off.



Let's draw the
(modified) braid group.

Let

$R = AB$

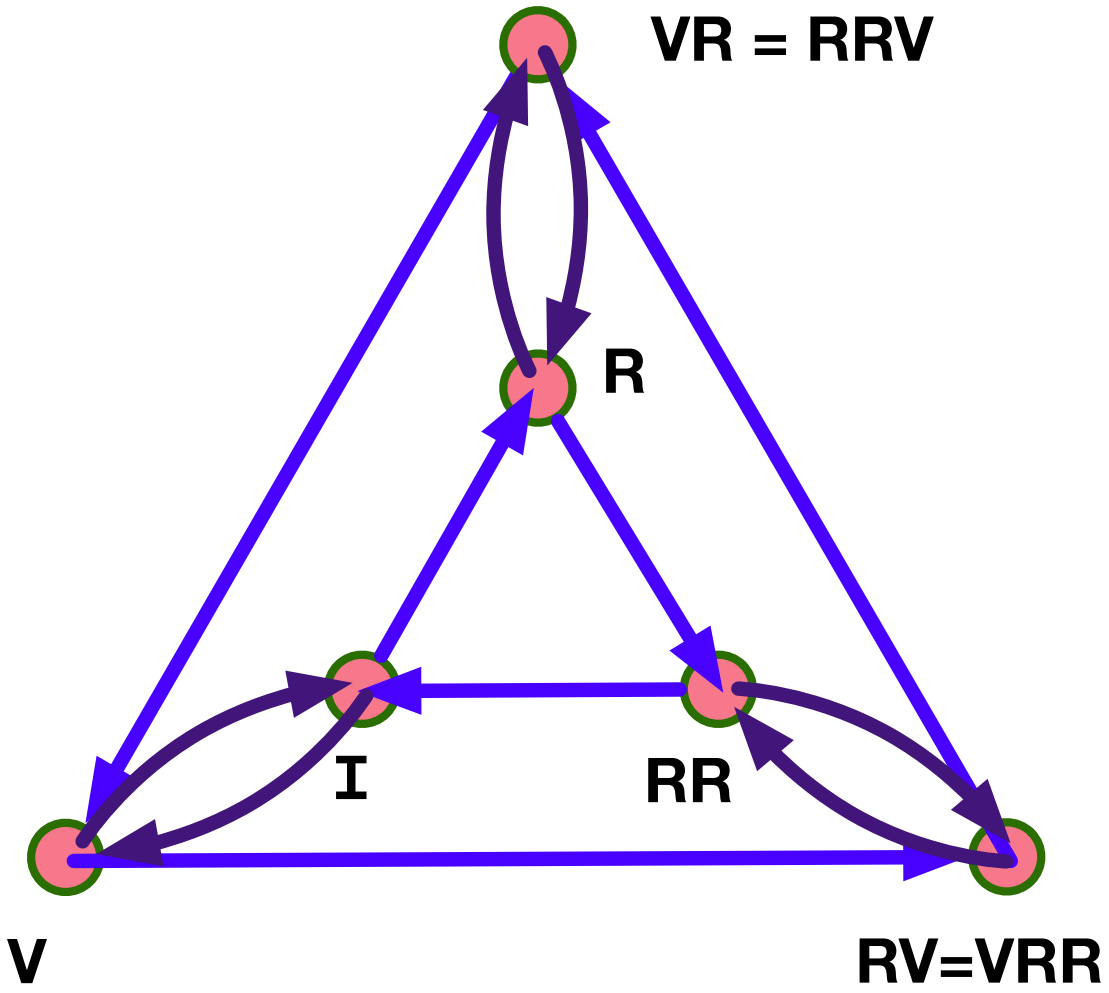
$V = ABA$

Then

$$RRR = (AB)(AB)(AB) = I$$

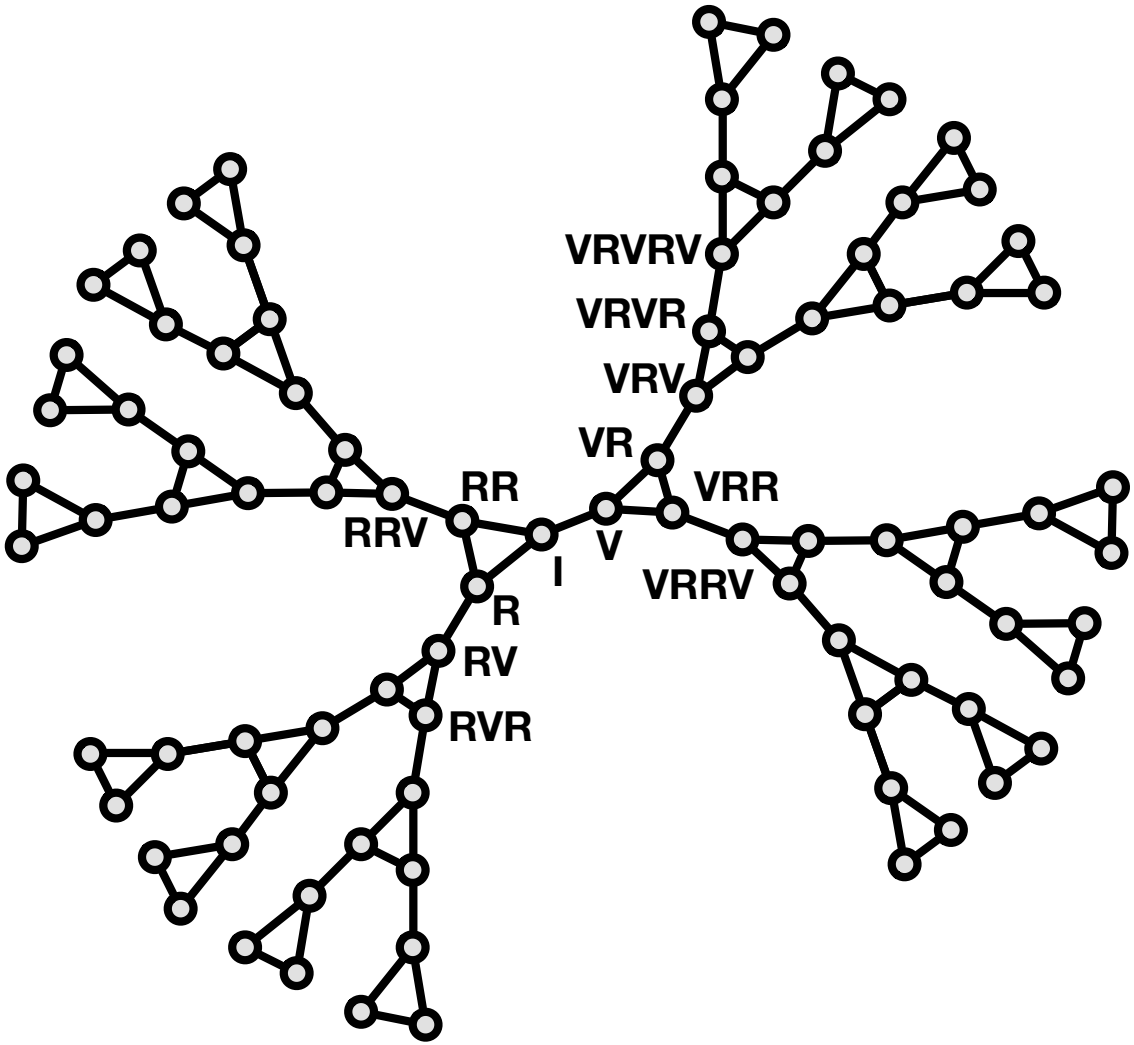
and

$$VV = (ABA)(ABA) = I.$$

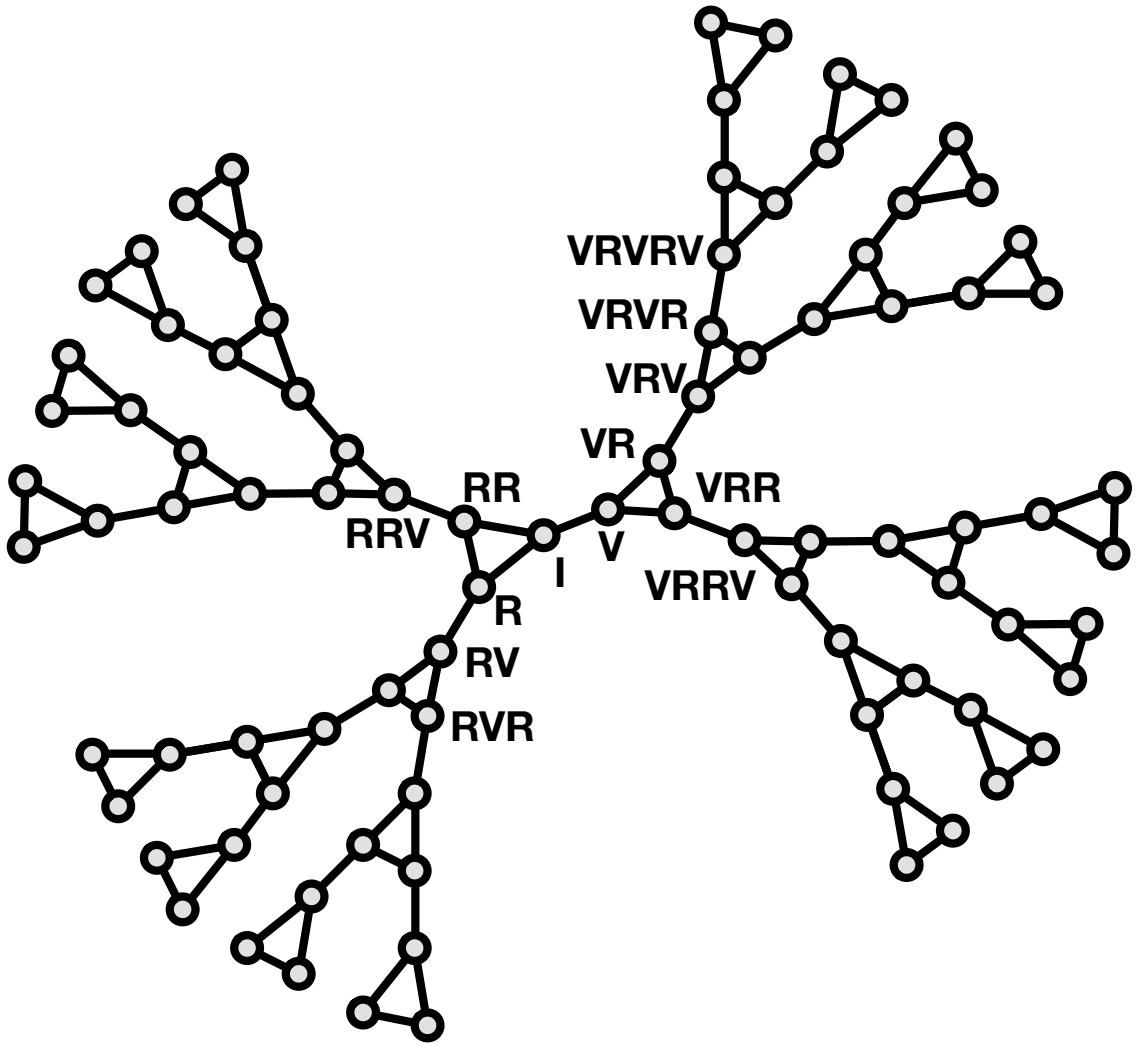


Recall this group: generators R and V, with

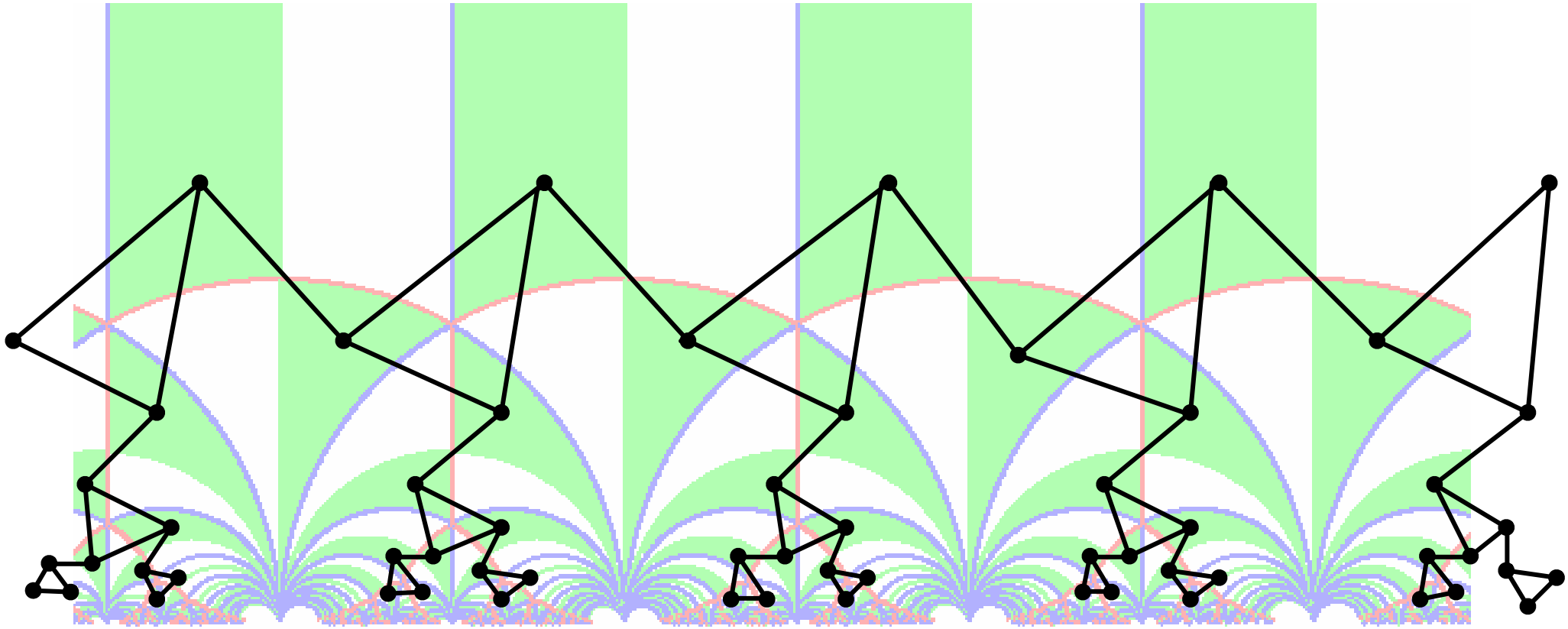
RRR = I, VV = I, **RV = VRR**



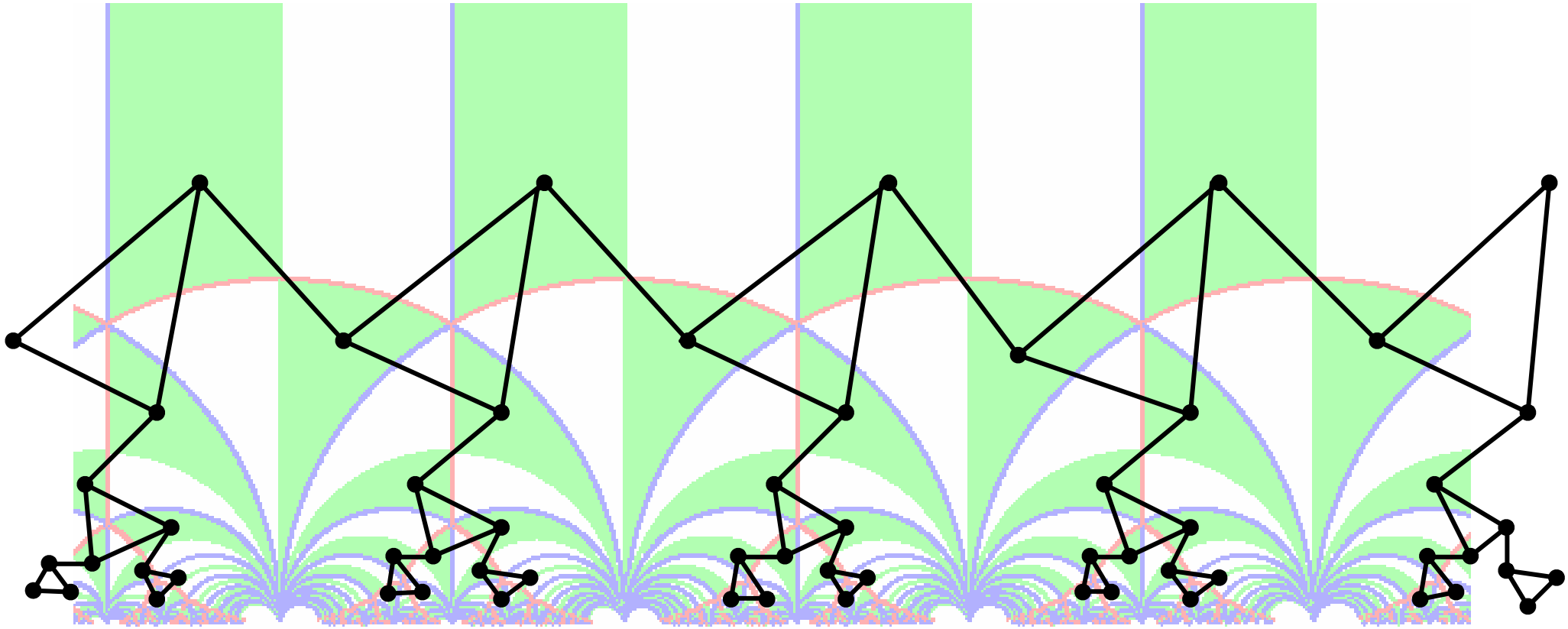
The group with generators R and V, with RRR = I and VV = I.



What kind of geometry does this look like?

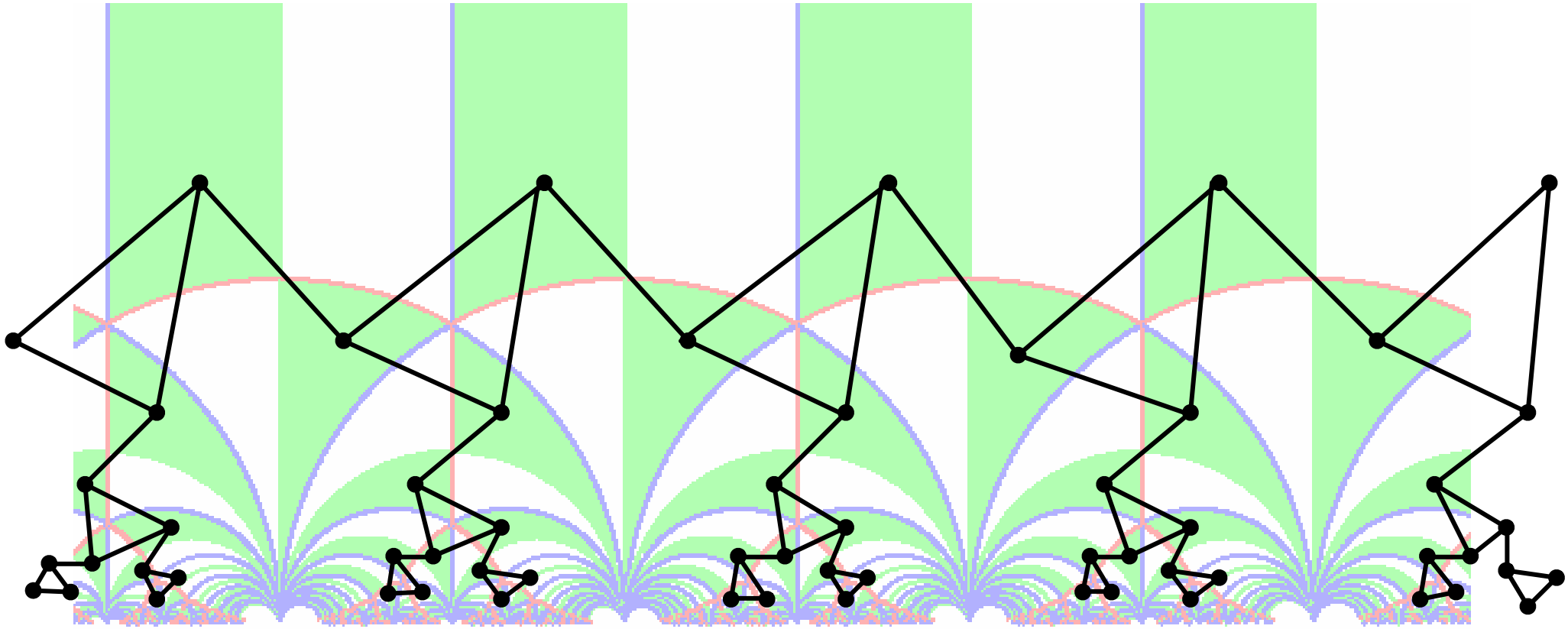


The modified braid group fits nicely in hyperbolic space.



To get the full braid group, imagine going "up" one level if you go around RRR or VV.

If you go around RRR or VV, you go up a level.



The full braid group looks like an infinitely tall parking garage with ramps up for every RRR and VV.

Google Map Space

What kind of geometry does Google Map Space have?

Points in Google Map Space:

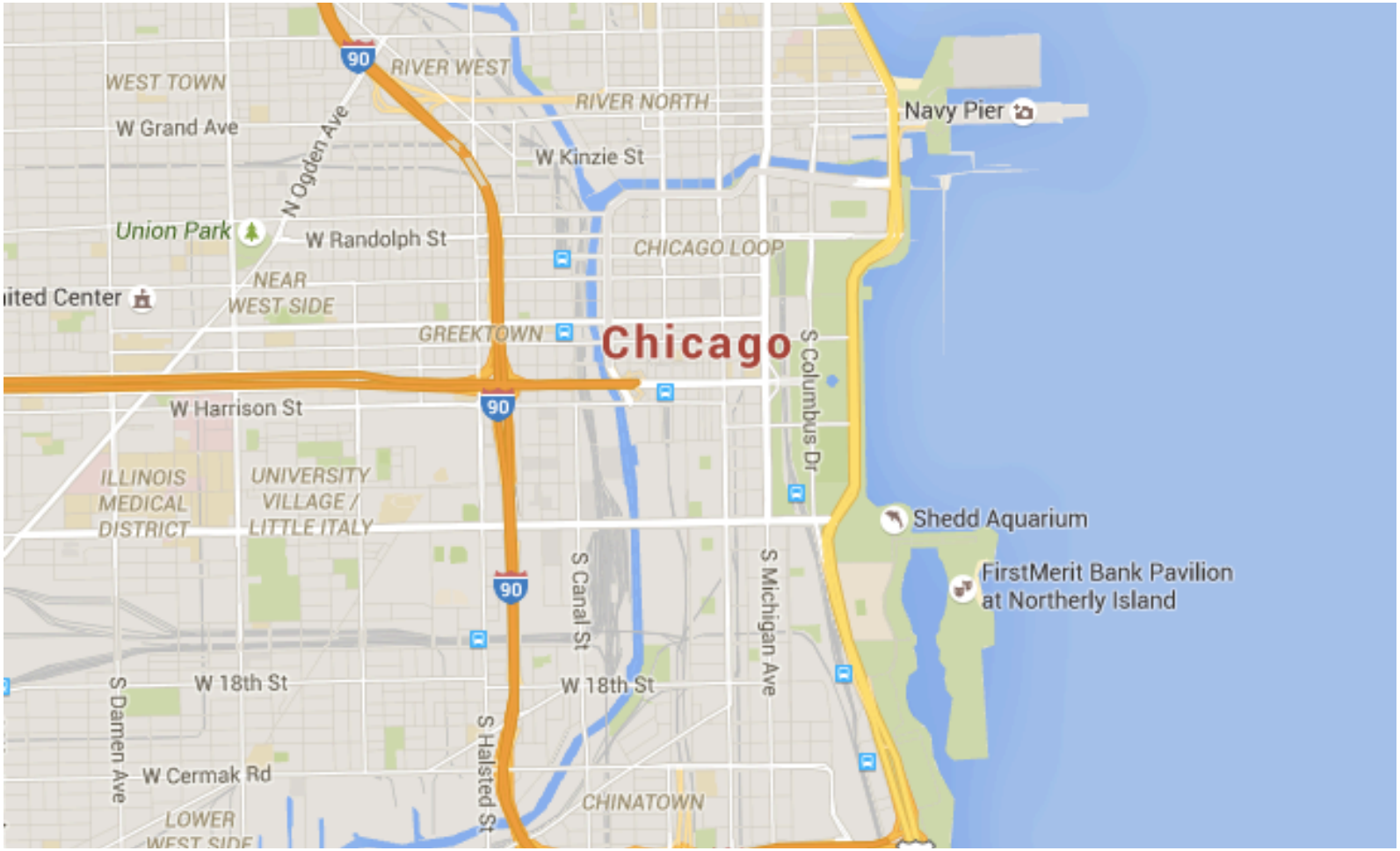
All possible Google map images.

Arrows:

I, O: zoom in or out

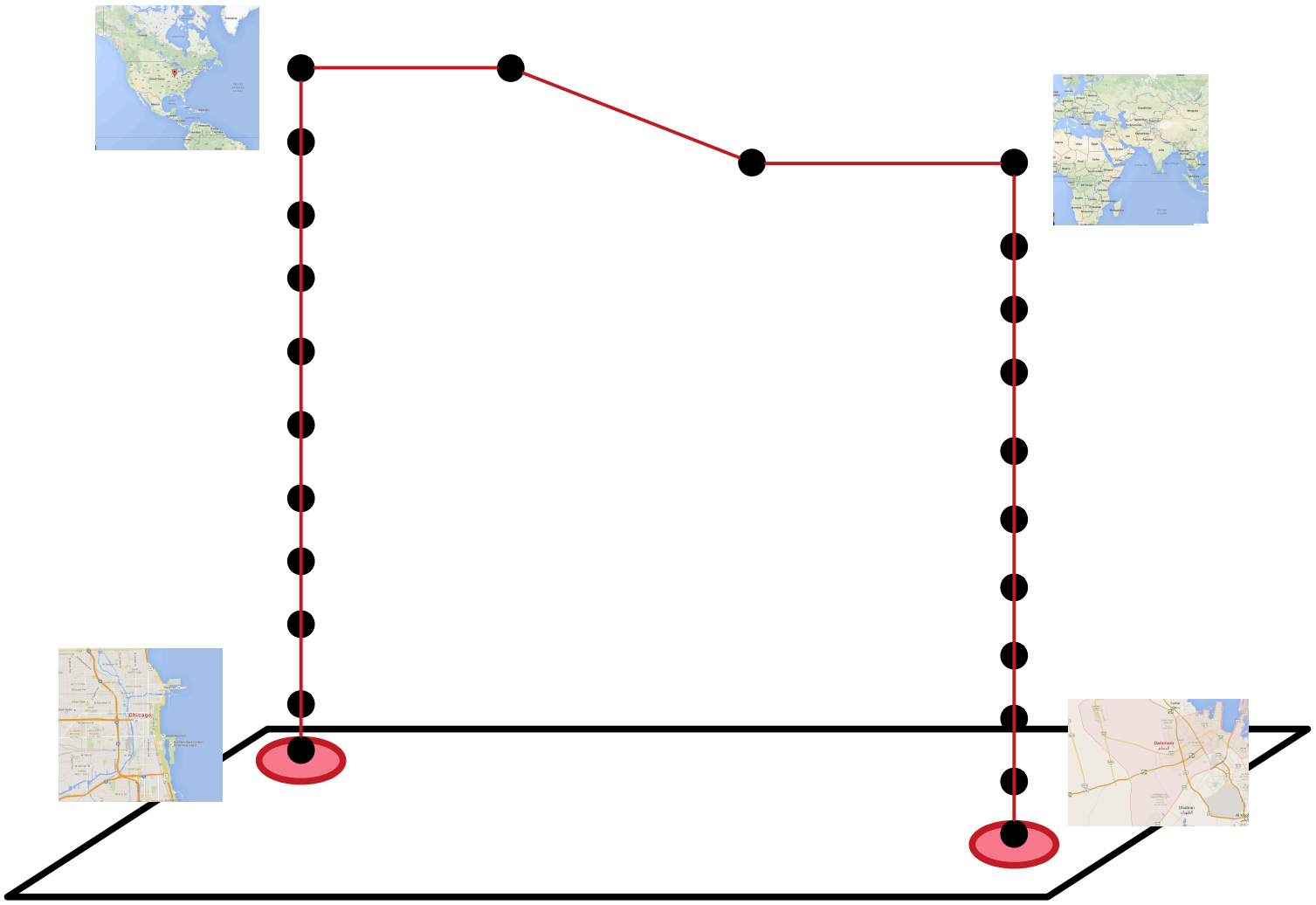
N, S, E, W: swipe north, south, east or west

What are the shortest paths in this space?

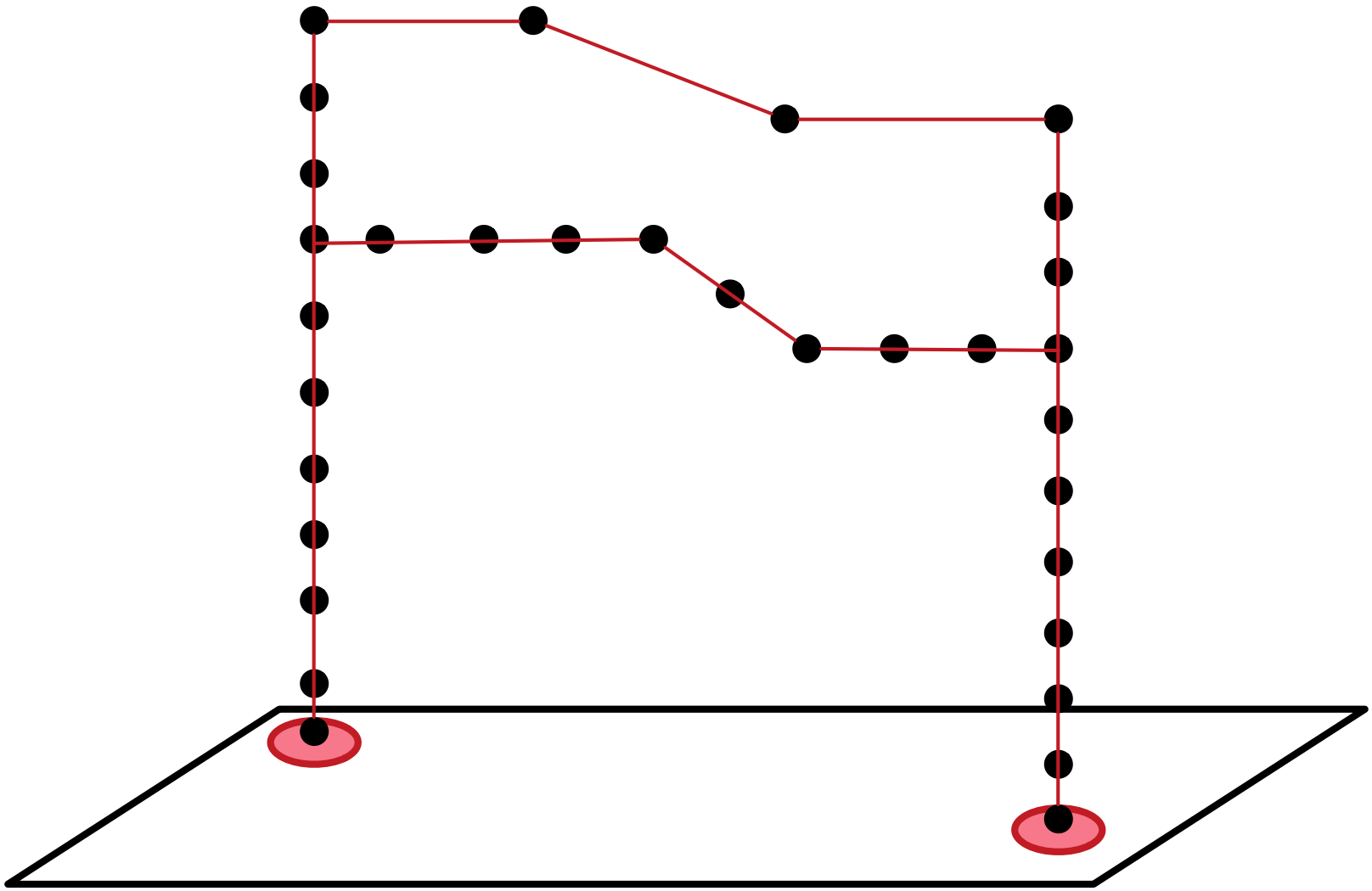


how do you go from a view of downtown Chicago

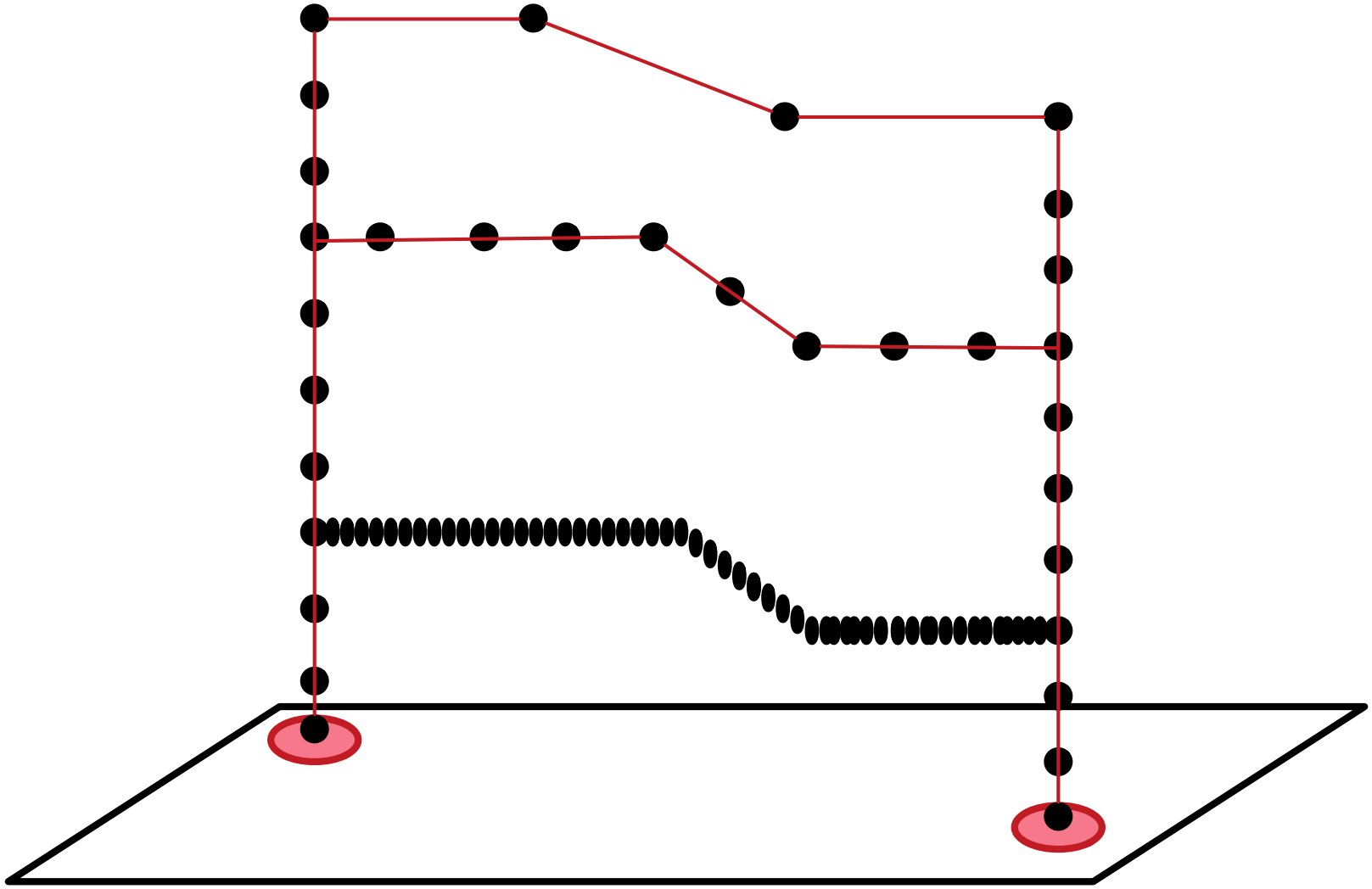
to a view of downtown Damman?



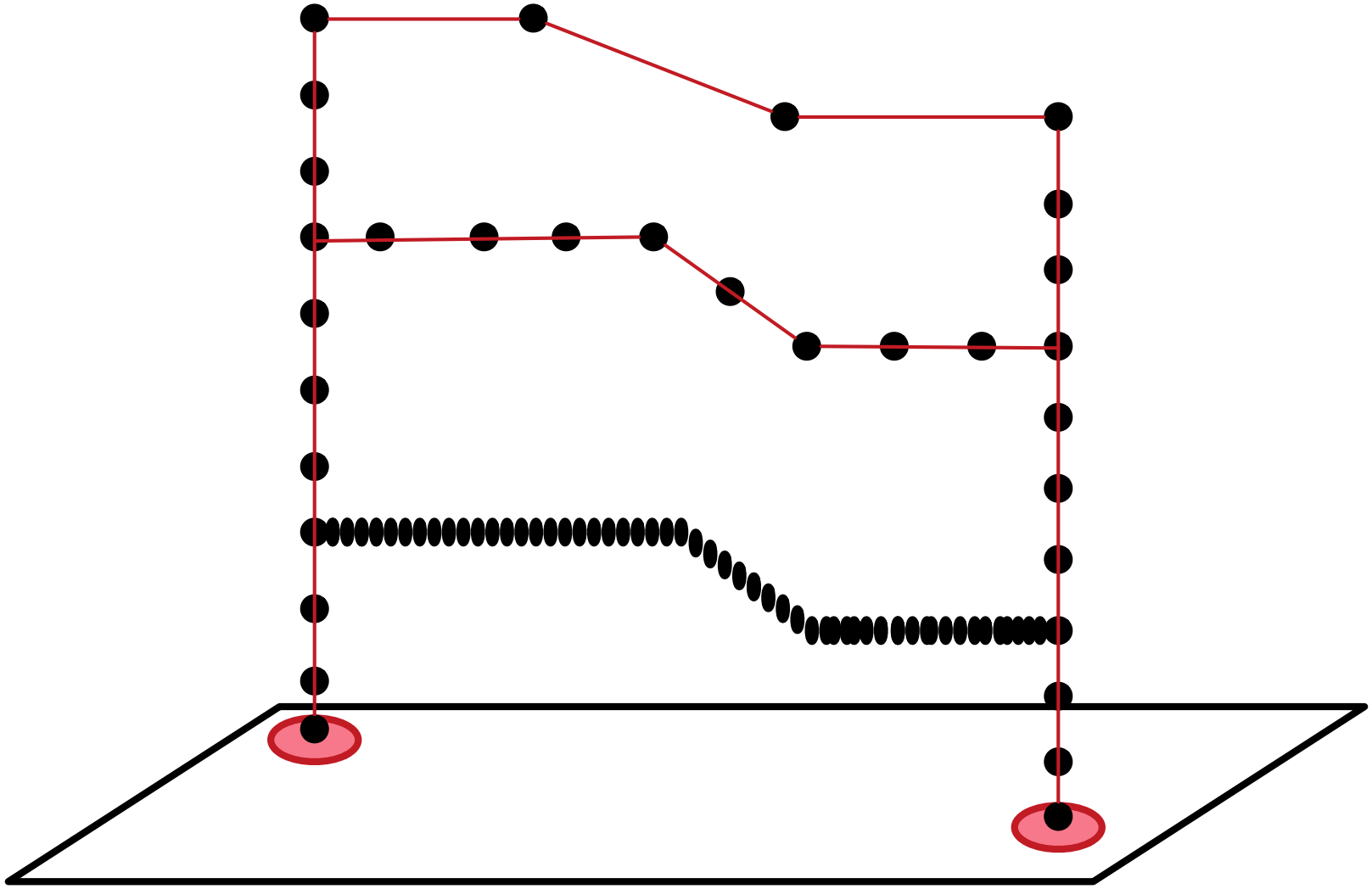
To get from Chicago to Dammam, zoom out 10 times, then swipe east and south, then zoom in 10 times.



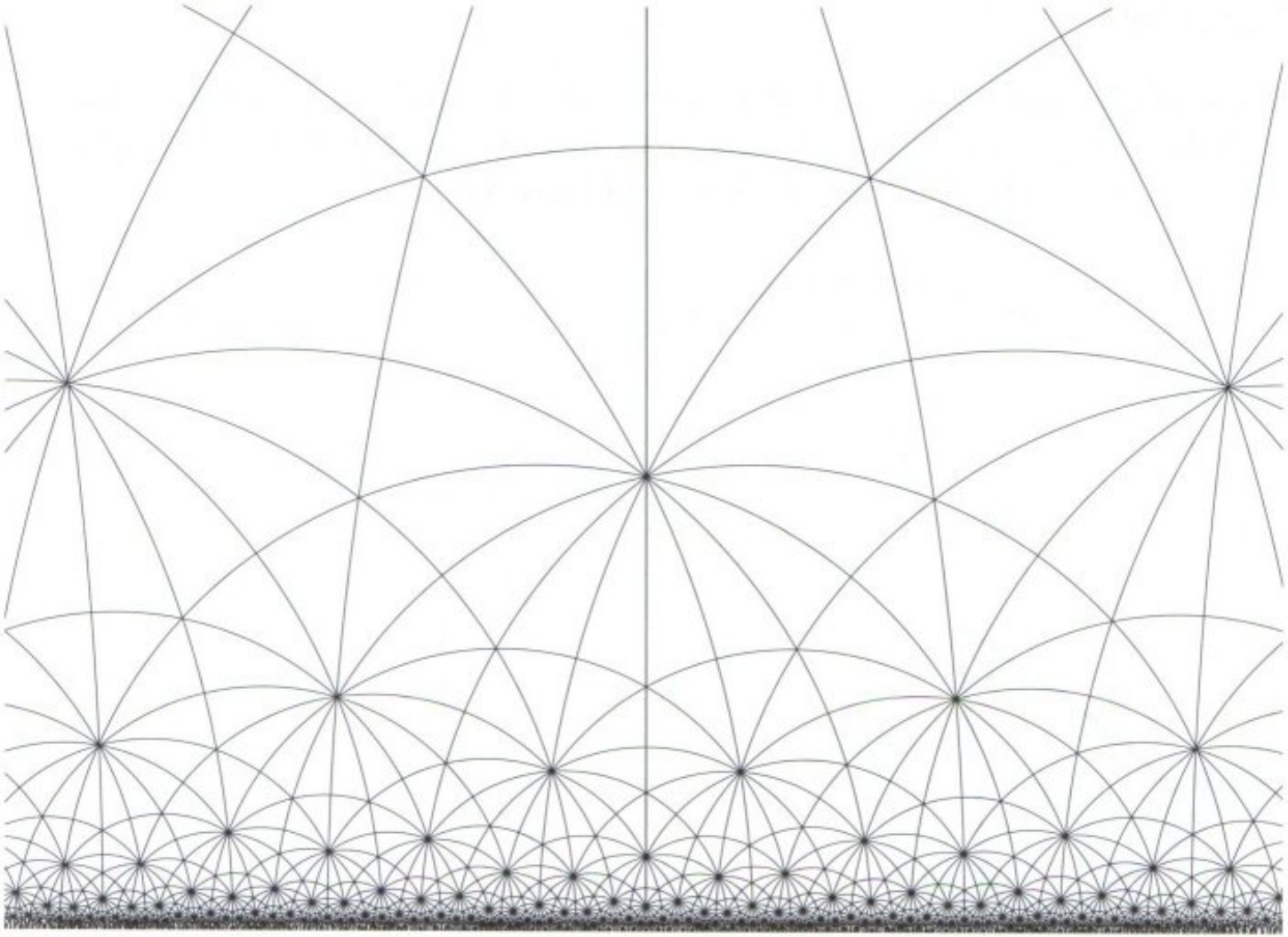
If we don't zoom out as much, it takes more swipes.



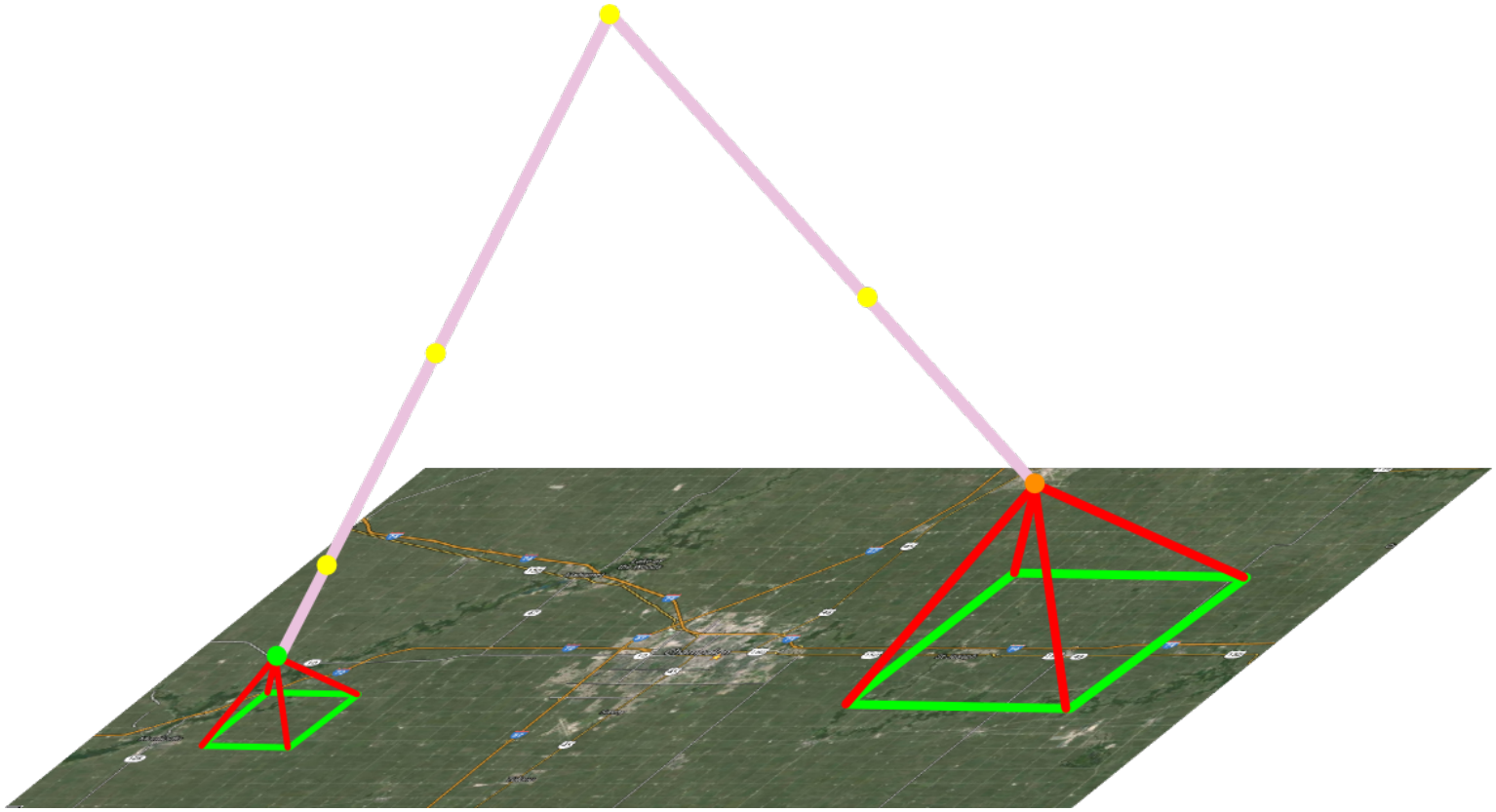
If we only zoom out 3 times,
it would take over 60 swipes.



What kind of geometry does this remind you of?



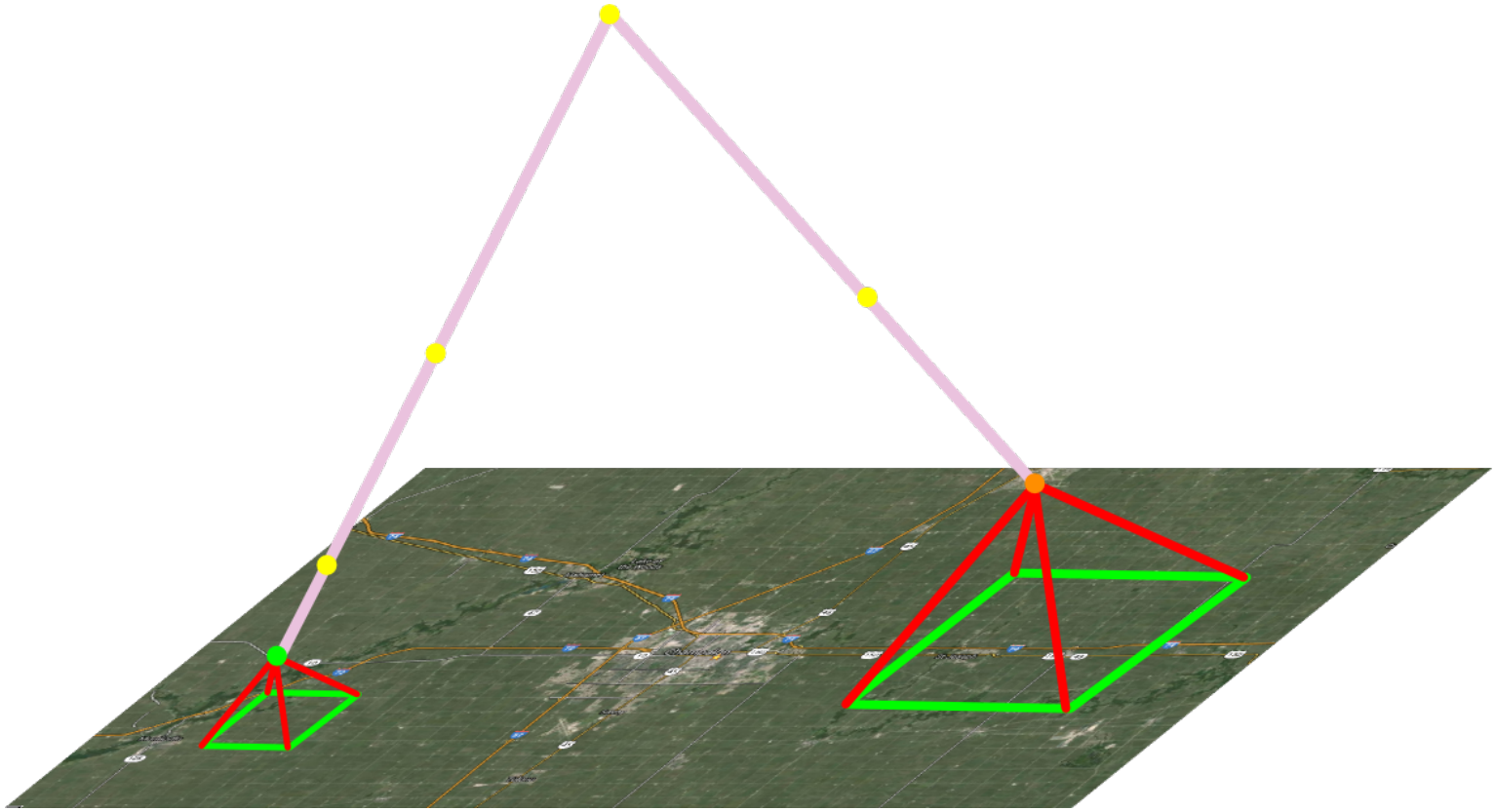
Recall: Upper Half Plane model of hyperbolic space



idea and picture: Yuliy Baryshnikov

Shortest paths in Google Map Space go up (zoom out), and then over, and then back down.

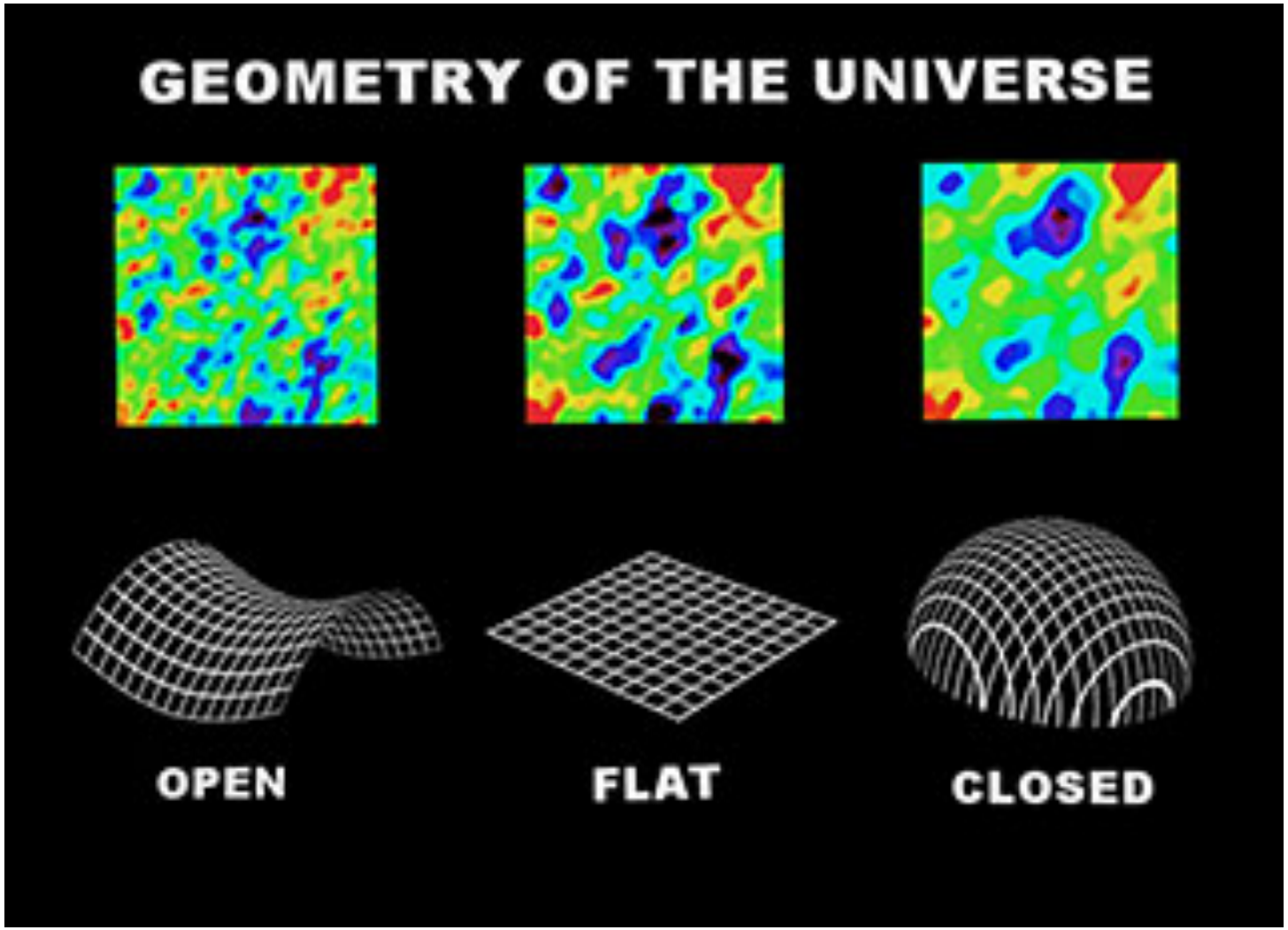
Upper Half Space Model of hyperbolic 3-space



idea and picture: Yuliy Baryshnikov

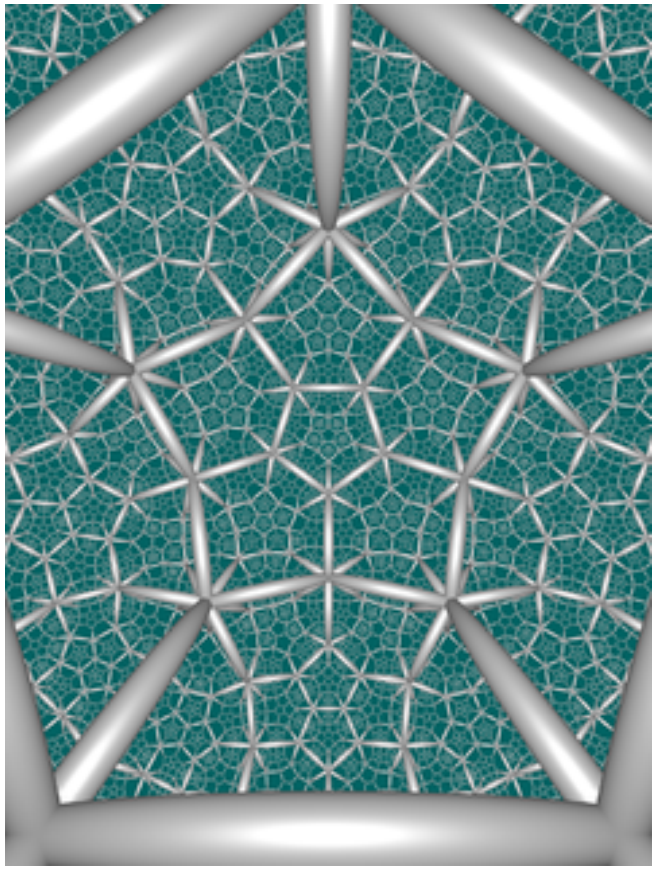
Google Map Space is (locally) like hyperbolic 3-space.

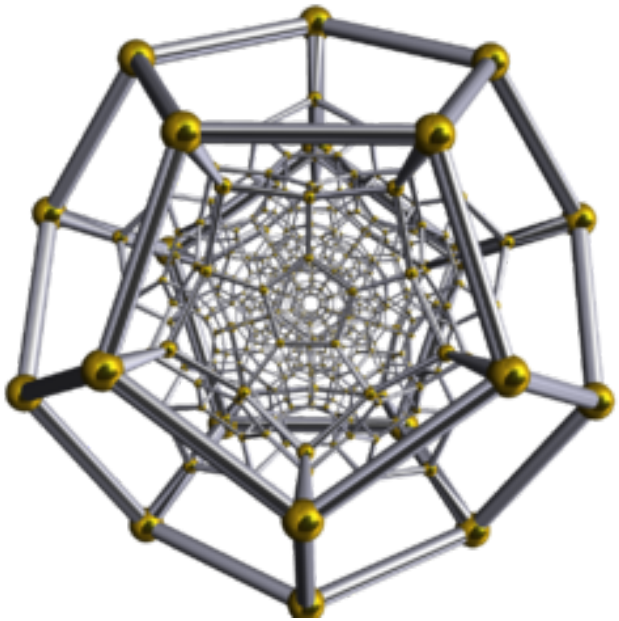
The Universe

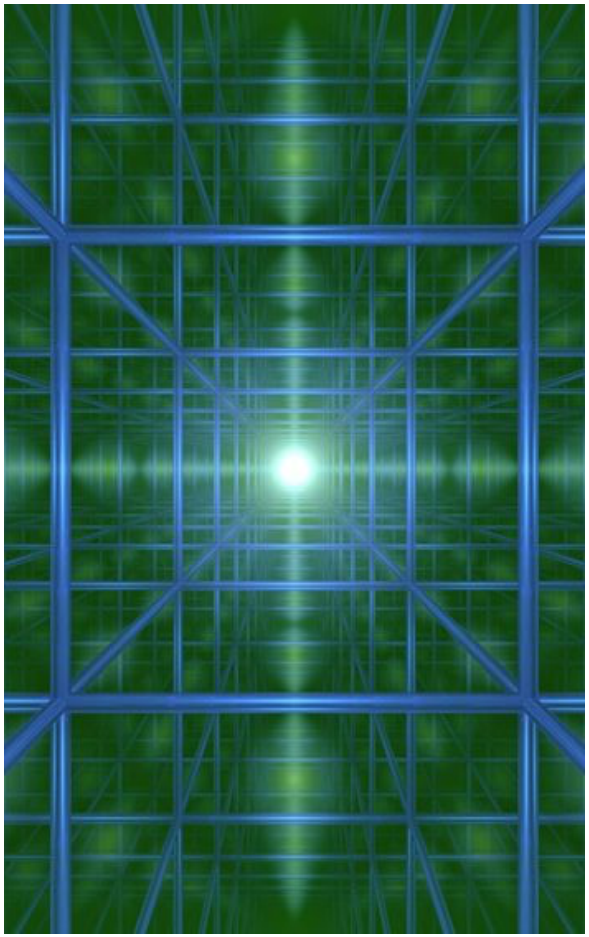


picture: NASA

What 3-dimensional geometry does the universe have?

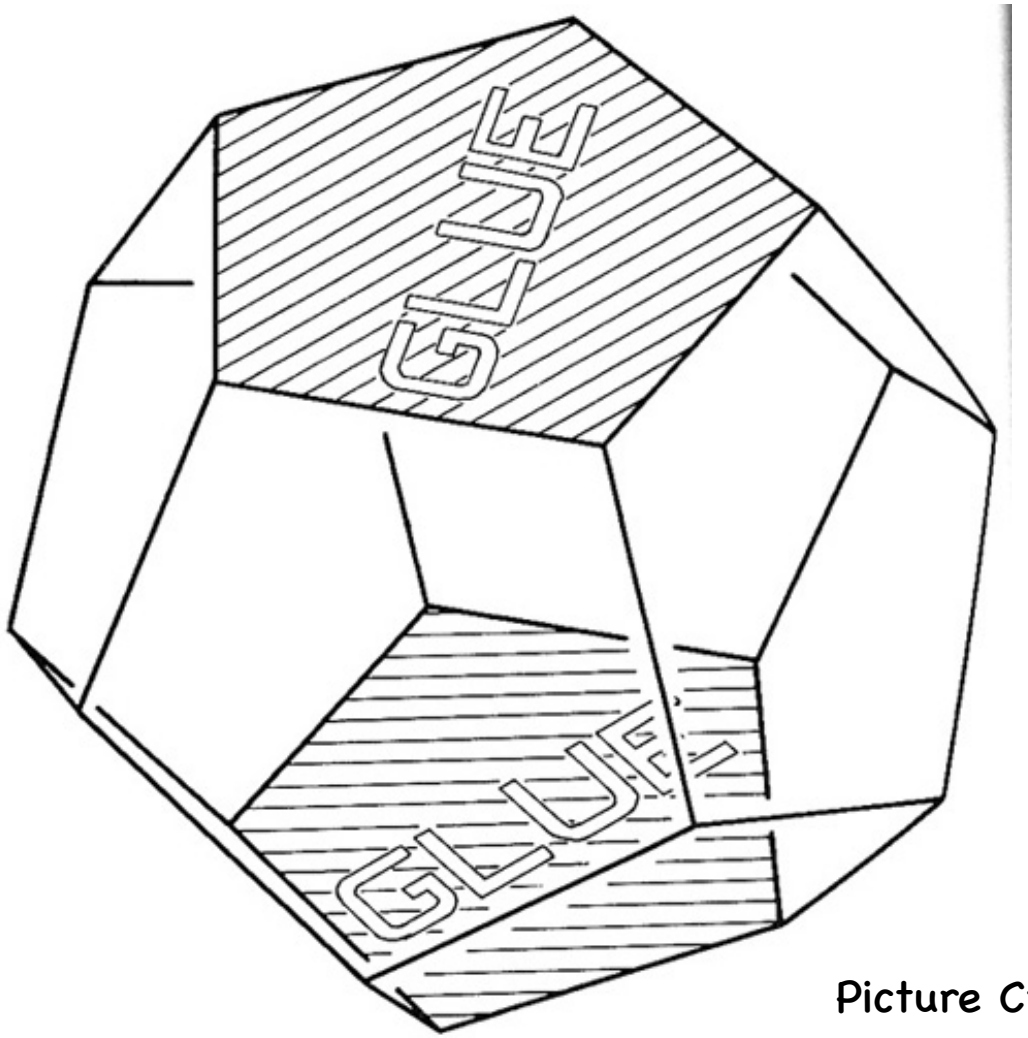






Is it hyperbolic,

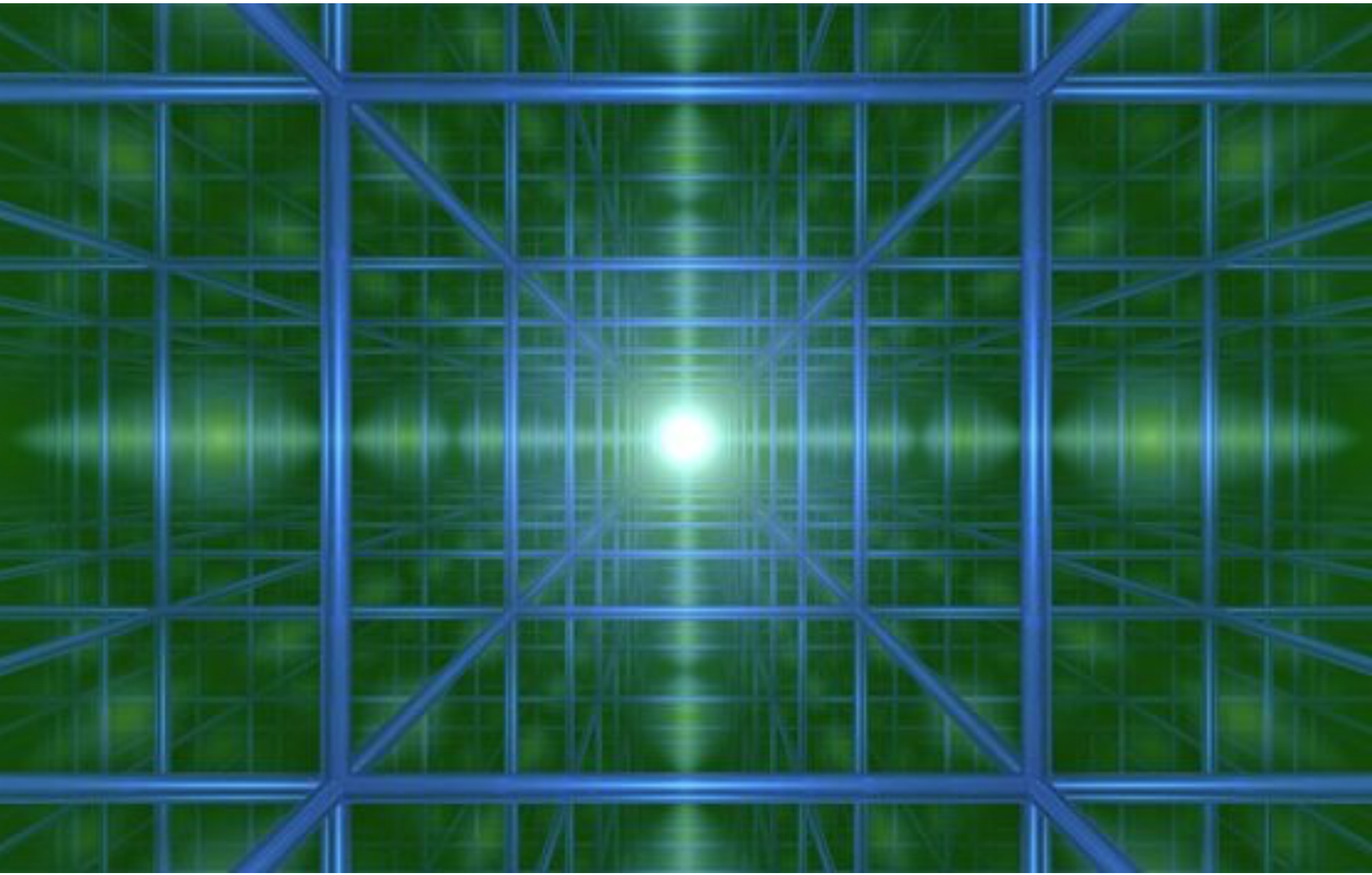
spherical, or Euclidean?



Picture Credit: Weeks 1985

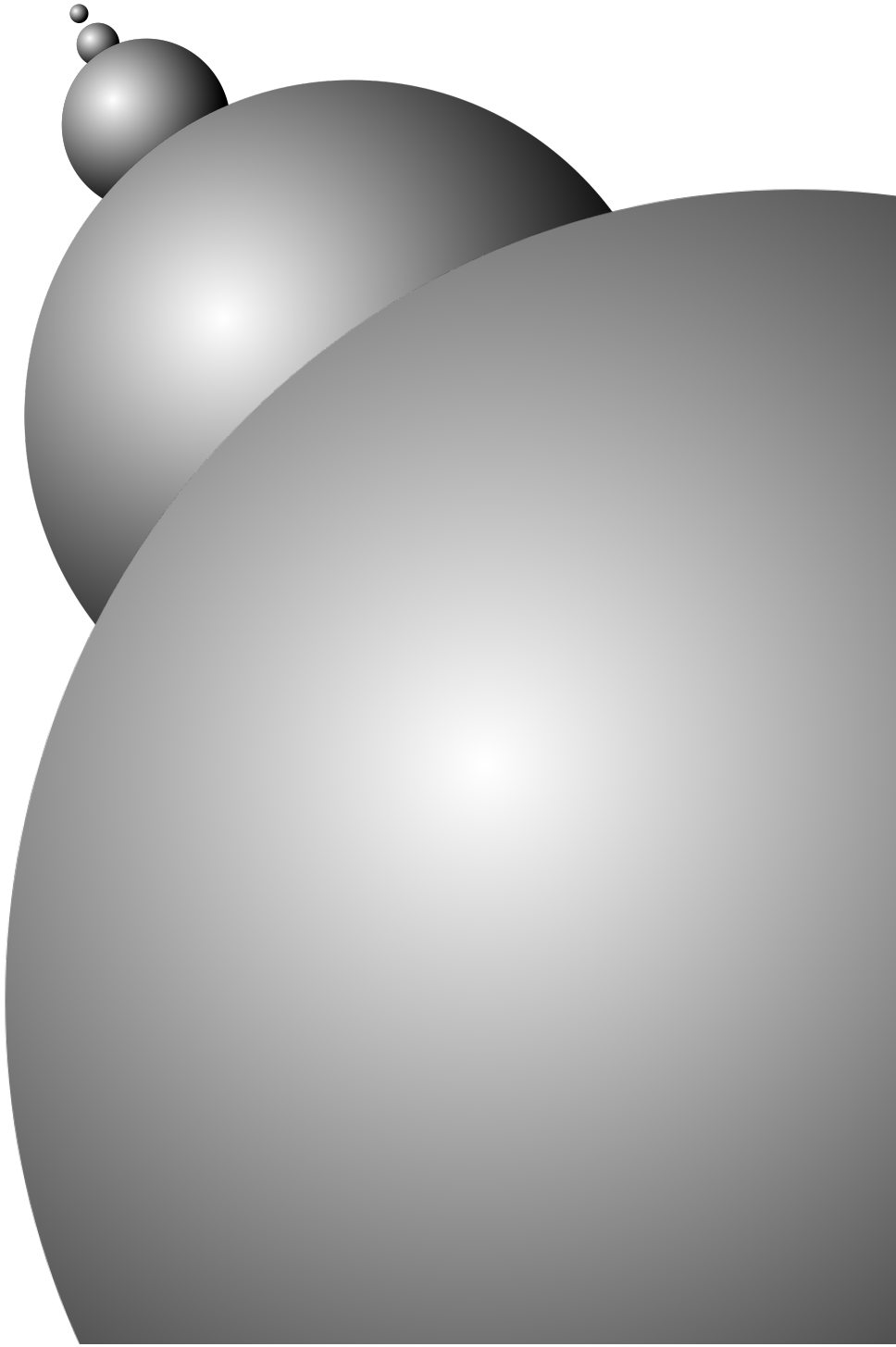
Some people thought the universe might be a Poincaré Dodecahedral Space, which has spherical geometry.

But recent measurements of background microwave radiation suggest that the universe is pretty close to flat. (Boomerang, WMAP, Planck)



But WHY is the geometry of the universe so close to flat?

One theory
is called
Inflation: an
extremely brief,
extremely rapid
expansion at the
start of time.



A really big sphere looks flat to an ant walking on it, as space does to us.

But whatever the reason, the data suggests that the universe is pretty close to flat.

But could it be a three-dimensional torus?

شكرا جزيلا !

Some cool links:

1. Vi Hart makes math cookies:
https://vimeo.com/147902577

2. https://publish.illinois.edu/ymb/2014/08/10/hyperbolic-geometry-of-google-maps/

3. http://map.gsfc.nasa.gov/universe/uni_shape.html

4. Thurston goes around a trefoil:
https://www.youtube.com/watch?v=IKSrBt2kFD4